Class - X Session 2022-23
Subject - Mathematics (Basic)
Sample Question Paper - 25

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A, B, C, D, and E.
2. Section A has 20 Multiple Choice Questions (MCQs) carrying 1 mark each.
3. Section B has 5 Short Answer-I (SA-I) type questions carrying 2 marks each.
4. Section C has 6 Short Answer-II (SA-II) type questions carrying 3 marks each.
5. Section D has 4 Long Answer (LA) type questions carrying 5 marks each.
6. Section E has 3 Case Based integrated units of assessment (4 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 2 marks, 2 Qs of 3 marks and 2 Questions of 5 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E.
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

Section A

1. In a $\triangle ABC$, AD is the bisector of $\angle BAC$. lt AB = 6 cm, AC = 5 cm and BD = 3 cm, then DC = **[1]**

   a) 11.3 cm b) None of these
   c) 3.5 cm d) 2.5 cm

2. Which of the following expressions is not a polynomial? **[1]**

   a) $5x^3 - 3x^2 - \sqrt{x} + 2$ b) $5x^3 - 3x^2 - x + \sqrt{2}$
   c) $5x^2 - \frac{2}{3}x + 2\sqrt{5}$ d) $\sqrt{5}x^3 - \frac{3}{5}x + \frac{1}{7}$

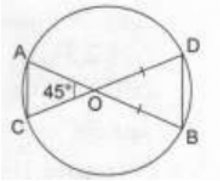
3. The value of c for which the pair of equations cx – y = 2 and 6x – 2y = 3 will have infinitely many solutions is **[1]**

   a) – 3 b) no value
   c) 3 d) –12

4. If x = a, y = b is the solution of the equations x – y = 2 and x + y = 4, then the values of a and b are, respectively **[1]**

   a) –1 and –3 b) 5 and 3
   c) 3 and 5 d) 3 and 1

5. In the given figure, O is the point of intersection of two chords AB and CD such that OB = OD and $\angle AOC = 45°$. Then, $\triangle OAC$ and $\triangle ODB$ are **[1]**



a) equilateral and similar
b) equilateral but not similar
c) isosceles but not similar
d) isosceles and similar

6. A die is thrown once. The probability of getting an even number is [1]

a) $\frac{1}{3}$
b) $\frac{5}{6}$
c) $\frac{1}{6}$
d) $\frac{1}{2}$

7. If $2\sin 2\theta = \sqrt{3}$ then θ= ? [1]

a) 45°
b) 90°
c) 60°
d) 30°

8. The mode of 4, 5, 6, 8, 5, 4, 8, 5, 6, x, 8 is 8. The value of x is [1]

a) 5
b) 6
c) 8
d) 4

9. The line segments joining the midpoints of the adjacent sides of a quadrilateral form [1]

a) a rhombus
b) a square
c) a parallelogram
d) a rectangle

10. If the LCM of a and 18 is 36 and the HCF of a and 18 is 2, then a = [1]

a) 1
b) 2
c) 4
d) 3

11. If the equation x^2 - ax + 1 = 0 has two distinct roots, then [1]

a) None of these
b) $|a| > 2$
c) $|a| = 2$
d) $|a| < 2$

12. The distance of the point (4, 7) from the y-axis is [1]

a) 11
b) 4
c) $\sqrt{65}$
d) 7

13. The arithmetic mean of 1, 2, 3, 4, ..., n is: [1]

a) $\frac{n-1}{2}$

b) $\frac{n(n+1)}{2}$

c) $\frac{n}{2}$

d) $\frac{n+1}{2}$

14. If $\sec\theta = \frac{25}{7}$ then $\sin\theta$ = ? **[1]**

a) $\frac{24}{7}$

b) $\frac{24}{25}$

c) $\frac{7}{24}$

d) none of these

15. A ladder 12 m long rests against a wall. If it reaches the wall at a height of $6\sqrt{3}$ m, then the angle of elevation is **[1]**

a) 60°

b) 30°

c) 75°

d) 45°

16. If angle between two radii of a circle is 130^o, the angle between tangents at ends of radii is : **[1]**

a) 70°

b) 90°

c) 60°

d) 50°

17. If in two triangles ABC and DEF, ∠A = ∠E, ∠B = ∠F, then which of the following is not true? **[1]**

a) $\frac{AB}{EF} = \frac{AC}{DE}$

b) $\frac{BC}{DF} = \frac{AC}{DE}$

c) $\frac{AB}{DE} = \frac{BC}{DF}$

d) $\frac{BC}{DF} = \frac{AB}{EF}$

18. Which of the following is a quadratic equation? **[1]**

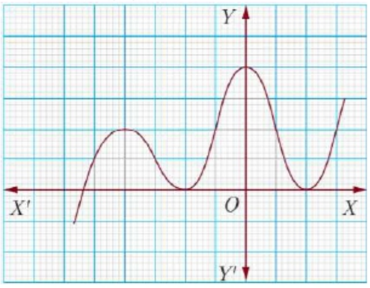
a) $x^3 - x^2 = (x-1)^3$

b) $x^2 + 2x + 1 = (4-x)^2 + 3$

c) $-2x^2 = (5-x)\left(2x - \frac{2}{5}\right)$

d) $(k+1)x^2 + \frac{3}{2}x - 5 = 0$, where k = – 1

19. **Assertion (A):** The graph y = f(x) is shown in figure, for the polynomial f(x). The number of zeros of f(x) is 3. **[1]**
Reason (R): The number of zero of the polynomial f(x) is the number of point of which f(x) cuts or touches the axes.



a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** Two identical solid cubes of side 5 cm are joined end to end. The total surface area of the resulting cuboid is 300 cm^2. **[1]**

Reason (R): Total surface area of a cuboid is 2(lb + bh + lh)

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Solve the following problem: $x^2 - 45x + 324 = 0$ **[2]**

22. Find the distance of C(-4, -6) points from the origin. **[2]**

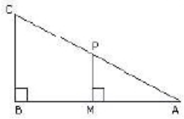
OR

Find the coordinates of a point A, where AB is the diameter of a circle whose centre is (2, -3) and B is (1, 4).

23. Express the HCF of 234 and 111 as 234x + 111y, where x and y are integers. **[2]**

24. Prove that: $(\sec^2\theta - 1)(1 - \operatorname{cosec}^2\theta) = -1$ **[2]**

25. In the given figure, ABC and AMP are two right-angled triangles, right angled at B and M respectively, prove that: **[2]**

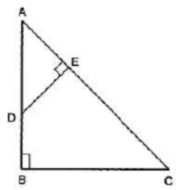


i. ABC ~ △ AMP

ii. $\frac{BC}{MP} = \frac{CA}{PA}$

OR

In Fig. if $AB \perp BC$ and $DE \perp AC$. Prove that △ABC ~ △AED.



Section C

26. Two numbers differ by 3 and their product is 504. Find the numbers. **[3]**

27. ABCD is a trapezium in which $AB || DC$. P and Q are points on sides AD and BC such that $PQ || AB$. If PD= 18, BQ = 35 and QC= 15, find AD. **[3]**

28. Point A lies on the line segment PQ joining P(6, -6) and Q (-4, -1) in such a way that $\frac{PA}{PQ} = \frac{2}{5}$. If the point A also lies on the line 3x + k(y + 1)=0, find the value of k. **[3]**

OR

In equilateral $\triangle$ABC , coordinates of points A and B are (2,0) and (5,0) respectively. Find the co-ordinates of the other two vertices.

29. Find the greatest number that will divide 445, 572 and 699 leaving remainders 4, 5 and 6 respectively. **[3]**

30. The pilot of an aircraft flying horizontally at a speed of 1200 km/hr. observes that the angle of depression of a point on the ground changes from 30° to 45° in 15 seconds. Find the height at which the aircraft is flying. **[3]**

OR

From a top of a building 100 m high the angle of depression of two objects are on the same side observed to be 45^{o} and 60^{o}. Find the distance between the objects.

31. Find the mean, median and mode of the following data: **[3]**

Class	0 - 20	20 - 40	40 - 60	60 - 80	80 - 100	100 - 120	120 - 140
Frequency	6	8	10	12	6	5	3

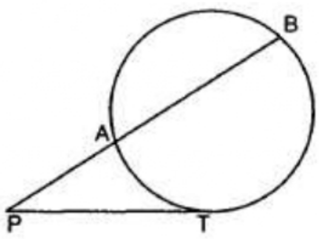
Section D

32. Solve the pairs of linear equation by the elimination method and the substitution method: $\frac{x}{2} + \frac{2y}{3} = -1\, and\, x - \frac{y}{3} = 3$ **[5]**

OR

The ratio of incomes of two persons is 11 : 7 and the ratio of their expenditures is 9 : 5. If each of them manages to save Rs 400 per month, find their monthly incomes.

33. In the given figure, PT is tangent to the circle at T. If PA = 4 cm and AB = 5 cm, find PT. **[5]**



34. A chord of a circle of radius 10cm subtends a right angle at the center. Find the area of the corresponding: (Use π = 3.14) **[5]**

i. minor sector

ii. major sector

iii. minor segment

iv. major segment

OR

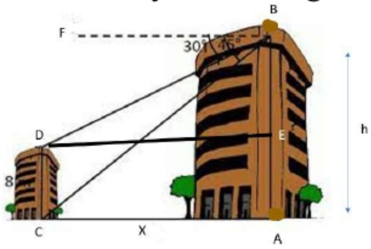
Find the difference of the areas of two segments of a circle formed by a chord of length 5 cm subtending angle of 90° at the centre.

35. The houses in a row are numbered consecutively from 1 to 49. Show that there exists a value of X such that sum of numbers of houses proceeding the house numbered X is equal to sum of the numbers of houses following X. **[5]**

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Basant and Vinod lives in a housing society in Dwarka, New Delhi. There are two building in their housing society. The first building is 8 meter tall. One day, both of them were just trying to guess the height of the other multi-storeyed building. Vinod said that it might be a 45 degree angle from the bottom of our building to the top of multi-storeyed building so the height of the building and distance from our building to this multi-storeyed building will be same. Then, both of them decided to estimate it using some trigonometric tools. Let's assume that the first angles of depression of the top and bottom of an 8 m tall building from top of a multi-storeyed building are 30^{o} and 45^{o}, respectively.



(i) Now help Vinod and Basant to find the height of the multistoried building.

(ii) Also, find the distance between two buildings.

(iii) Find the distance between top of multistoried building and bottom of first building.

OR

Find the distance between top of multistoried building and top of first building.

37. **Read the text carefully and answer the questions:** **[4]**

Elpis Technology is a TV manufacturer company. It produces smart TV sets not only for the Indian market but also exports them to many foreign countries. Their TV sets have been in demand every time but due to the Covid-19 pandemic, they are not getting sufficient spare parts, especially chips to accelerate the production. They have to work in a limited capacity due to the lack of raw materials.

(i) They produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find an increase in the production of TV every year.

(ii) They produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find in which year production of TV is 1000.

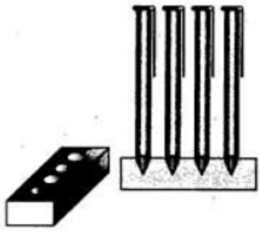
(iii) They produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find the production in the 10th year.

OR

They produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find the total production in first 7 years.

38. **Read the text carefully and answer the questions:** **[4]**

A carpenter in the small town of Bareilly used to make and sell different kinds of wood items like a rectangular box, cylindrical pen stand, and cuboidal pen stand. One day a student came to his shop and asked him to make a pen stand with the dimensions as follows:
A pen stand should be in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid should be 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm.



(i) The volume of the cuboidal part.

(ii) The volume of wood in the entire stand.

(iii) Total volume of conical depression.

OR

If the cost of wood used is ₹10 per cm^3, then the total cost of making the pen stand.

SOLUTION

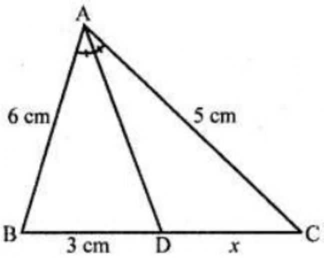
Section A

1. **(d)** 2.5 cm

   Explanation:

   In $\triangle ABC$, AD is the bisector of $\angle BAC$

   AB = 6 cm, AC = 5 cm, BD = 3 cm

   Let DC = x

   In $\triangle ABC$

   $\because$ AD is the bisector of $\angle A$

   $$\therefore \frac{AB}{AC} = \frac{BD}{DC} \Rightarrow \frac{6}{5} = \frac{3}{x}$$

   $$\Rightarrow x = \frac{3 \times 5}{6} = \frac{5}{2} = 2.5$$

   $\therefore$ DC = 2.5cm

2. **(a)** $5x^3 - 3x^2 - \sqrt{x} + 2$

   Explanation: $5x^3 - 3x^2 - \sqrt{x} + 2$ is not a polynomial because each term of a polynomial should be a product of a constant and one or more variable raised to a positive, zero or integral power. Here $\sqrt{x}$ does not satisfy the condition of being a polynomial.

3. **(b)** no value

   Explanation: Condition for infinitely many solutions

   $$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \ldots\text{(i)}$$

   The given lines are cx - y - 2 = 0 and 6x - 2y - 3 = 0;

   Comparing with standard form we get,

   $a_1 = c$, $b_1 = -1$, $c_1 = -2$

   and $a_2 = 6$, $b_2 = -2$, $c_2 = -3$

   From Eq. (i), we have

   c/6 = -1/-2

   Also c/6 = -2/-3

   Solving, we get, c = 3 and c = 4.

   Since, c has different values and so it's not possible.

   Hence, for no value of c the pair of equations will have infinitely many solutions.

4. **(d)** 3 and 1
Explanation: Given equations are:
x - y = 2 and
x + y = 4;
Adding them, we get
2x = 6
x = 3;
Subtracting them, we get
2y = 2
y = 1;
So, a = 3 and b = 1 is the solution of the equations.
5. **(d)** isosceles and similar
Explanation: In the given figure, O is the point of intersection of two chords AB and CD.
OB = OD and ∠AOC = 45°
∠B = ∠D (Angles opposite to equal sides)
∠A = ∠D, ∠C = ∠B (Angles in the same segment)
and ∠AOC = ∠BOD = 45° each
△OAC ~ △ODB (AAA axiom)
OA = OC (Sides opposite to equal angles)
△OAC and △ODB are isosceles and similar.
6. **(d)** $\frac{1}{2}$
Explanation: Number of all possible outcomes = 6.
Even numbers are 2,4, 6. Their number is 3.
$\therefore$ P (getting an even number) = $\frac{3}{6} = \frac{1}{2}$
7. **(d)** 30°
Explanation: We have, $2\sin 2\theta = \sqrt{3} \Rightarrow \sin 2\theta = \frac{\sqrt{3}}{2} = \sin 60^\circ$
$\Rightarrow 2\theta = 60^\circ \Rightarrow \theta = 30^\circ$
8. **(c)** 8
Explanation: Here Observations 5 and 8 have the same frequency, i.e., 3.
For 8 be the mode of the data, it should have the maximum frequency.
$\therefore$ 8 should repeat itself at least one more.
$\Rightarrow$ x should be 8.
9. **(c)** a parallelogram
Explanation: The line segments joining the midpoints of the adjacent sides of a quadrilateral form a parallelogram.
10. **(c)** 4
Explanation: LCM (a, 18) = 36
HCF (a, 18) = 2
We know that the product of numbers is equal to the product of their HCF and LCM.
Therefore,

$18a = 2(36)$

$a = \frac{2(36)}{18}$

$a = 4$

11. **(b)** $|a| > 2$

Explanation: In the equation $x^2 - ax + 1 = 0$

$a = 1, b = -a, c = 1$

$D = b^2 - 4ac = (-a)^2 - 4 \times 1 \times 1 = a^2 - 4$

Roots are distinct

$D > 0$

$\Rightarrow a^2 - 4 > 0$

$\Rightarrow a^2 > 4$

$\Rightarrow a^2 > (2)^2$

$\Rightarrow |a| > 2$

12. **(b)** 4

Explanation: The distance of the point (4, 7) from y-a x is = 4

13. **(d)** $\frac{n+1}{2}$

Explanation: According to question,

Arithmetic Mean = $\frac{1+2+3+\ldots+n}{n}$

$= \frac{\frac{n(n+1)}{2}}{n}$

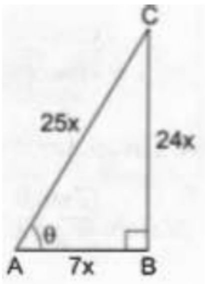
$= \frac{n+1}{2}$

14. **(b)** $\frac{24}{25}$

Explanation: $\sec\theta = \frac{AC}{AB} = \frac{25}{7} = \frac{25x}{7x} \Rightarrow AC = 25x$ and $AB = 7x$

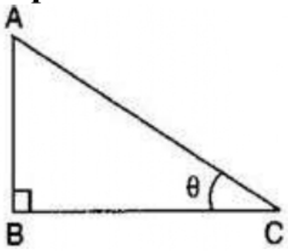
$\therefore BC^2 = AC^2 - AB^2 = 625x^2 - 49x^2 = 576x^2 \Rightarrow BC = 24x$

$\therefore \sin\theta = \frac{BC}{AC} = \frac{24x}{25x} = \frac{24}{25}$



15. **(a)** 60 °

Explanation: Let the ladder be AC of the length 12 m.



Then the height AB is $6\sqrt{3}$meter.

$\therefore \sin\theta = \frac{AB}{AC}$

$\Rightarrow \sin\theta = \frac{6\sqrt{3}}{12}$

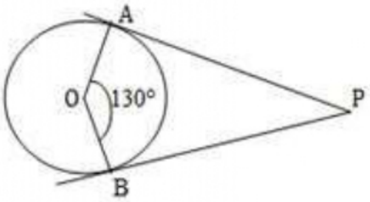
$\Rightarrow \sin\theta = \frac{\sqrt{3}}{2}$

$\Rightarrow \sin\theta = \sin 60°$

$\Rightarrow \theta = 60°$

16. **(d)** 50 °

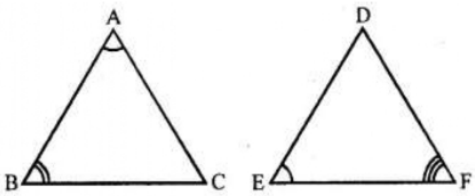
Explanation: If the angle between two radii of a circle is 130^o, the angle between tangents at ends of radii is$\angle APB = 50°$. Because the angle between the two tangents drawn from an external point to a circle is supplementary of the angle between the radii of the circle through the point of contact.



17. **(c)** $\frac{AB}{DE} = \frac{BC}{DF}$

Explanation: In two triangles ABC and DEF

$\angle A = \angle E, \angle B = \angle F$



$\therefore \frac{BC}{DF} = \frac{AC}{DE} = \frac{AB}{EF}$

$\therefore \frac{AB}{DE} = \frac{BC}{DF}$ is not true

18. **(a)** $x^3 - x^2 = (x-1)^3$

Explanation: In equation $x^3 - x^2 = (x-1)^3$

$\Rightarrow x^3 - x^2 = x^3 - 1 - 3x^2 + 3x$

$\Rightarrow -x^2 + 3x^2 - 3x + 1 = 0$

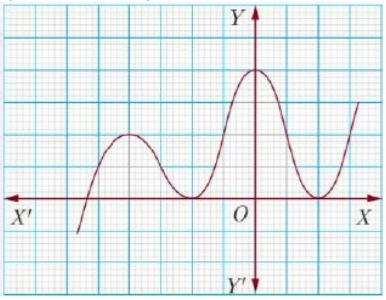
$\Rightarrow 2x^2 - 3x + 1 = 0$

It is a quadratic equation as its degree is 2.

19. **(c)** A is true but R is false.

Explanation:

As the number of zeroes of polynomial f(x) is the number of points at which f(x) cuts (intersects) then x-axis and number of zero in the given fig. is 3.



So, A is correct but R is not correct.

20. **(d)** A is false but R is true.

Explanation: A is false but R is true.

Section B

21. $x^2 - 45x + 324 = 0$

$\Rightarrow x^2 - 36x - 9x + 324 = 0 \Rightarrow x(x - 36) - 9(x - 36) = 0$

$\Rightarrow (x - 9)(x - 36) \Rightarrow x = 9, 36$

22. The given point is C(-4, -6) and let O(0,0) be the origin

Then, $CO = \sqrt{(-4-0)^2 + (-6-0)^2}$

$= \sqrt{16 + 36} = \sqrt{52} = 2\sqrt{13}$ *units*

OR

We want to find coordinates of point A. AB is the diameter and coordinates of centerare (2, -3) and, coordinates of point B are (1, 4).

Let coordinates of point A are (x, y). Using section formula, we get

$2 = \frac{x+1}{2}$

$\Rightarrow 4 = x + 1$

$\Rightarrow x = 3$

Using section formula, we get

$-3 = \frac{4+y}{2}$

$\Rightarrow 4 + y = -6$

$\Rightarrow y = -6 - 4 = -10$

Therefore, Coordinates of point A are (3, −10).

23. Given numbers are 234 and 111

Using Euclid's division lemma , we get

$234 = 111 \times 2 + 12$

Now $111 = 12 \times 9 + 3$

and $12 = 3 \times 4 + 0$

HCF = 3

As per given condition

$3 = 234x + 111y$

$\Rightarrow 3 - 234x = 111y$

$\Rightarrow \dfrac{3-234x}{111} = y$

Taking x = - 9, we get

$\dfrac{3-234(-9)}{111} = y$

Therefore, y = 19.

24. We have,

L.H.S = $(\sec^2\theta - 1)(1 - \operatorname{cosec}^2\theta)$

$= (1 + \tan^2\theta - 1)[1 - (1 + \cot^2\theta)]$ [$\because$ $\sec^2\theta = 1 + \tan^2\theta$ and $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$]

$= \tan^2\theta(-\cot^2\theta)$

$= \tan^2\theta.\left(\dfrac{-1}{\tan^2\theta}\right)$ [$\because$ $\cot\theta = \dfrac{1}{\tan\theta}$]

= -1 = R.H.S.

therefore, $(\sec^2\theta - 1)(1 - \operatorname{cosec}^2\theta) = -1$

Hence proved

25. In $\triangle$ABC and $\triangle$AMP,

$\angle B = \angle M$ (Each 90^o)

$\angle A = \angle A$ (common)

$\therefore$ $\triangle$ S are similar

i.e., $\triangle ABC \sim \triangle AMP$

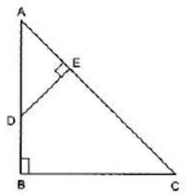
$\therefore \dfrac{BC}{MP} = \dfrac{CA}{PA}$

OR

Given: A triangle ABC in which $AB \perp BC$ and $DE \perp AC$.

To Prove: $\triangle ABC \sim \triangle AED$.

Proof: In $\triangle$'s ABC and AED, we have



$\angle ABC = \angle AED = 90^o$

$\angle BAC = \angle EAD$ (Each equal to $\angle A$]

Therefore, by AA-criterion of similarity, we obtain $\triangle ABC \sim \triangle AED$.

Section C

26. Sol : Let the required number be x and x + 3. Then,
according to given question we have,

$x \times (x + 3) = 504$

$\Rightarrow x^2 + 3x = 504$

$\Rightarrow x^2 + 3x - 504 = 0$

$\Rightarrow x^2 + 24x - 21x - 504 = 0$

$\Rightarrow x(x + 24) - 21(x + 24) = 0$

$\Rightarrow (x + 24)(x - 21) = 0$

$\Rightarrow x + 24 = 0$ or $x - 21 = 0$

$\Rightarrow x = - 24$ or $x = 21$

Case I: When x = -24

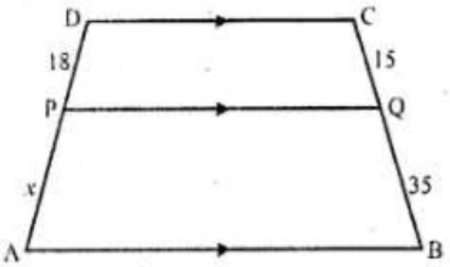
$\therefore x + 3 = -24 + 3 = -21$

Case II: When x = 21

$\therefore x + 3 = 21 + 3 = 24$

Hence, the numbers are -21, -24 or 21, 24.

27.



In trapezium ABCD.

$AB \parallel DC$

P and Q are points on AD and BC respectively such that
PQ∥BC, PD = 18, BQ = 35, QC = 15

Let PD = x

$\because DC \parallel AB \parallel PQ$

$\therefore \frac{DP}{PA} = \frac{CQ}{QB}$

$\Rightarrow \frac{18}{x} = \frac{15}{35}$

$\Rightarrow x = \frac{18 \times 35}{15} = 42$

$\therefore AD = AP + PD = 42 + 18 = 60$

28. Let the coordinates of A be (x, y) which lies on line joining P(6, -6) and Q (-4, -1)

such that $\frac{PA}{PQ} = \frac{2}{5}$

$\Rightarrow \frac{PA}{PQ - PA} = \frac{2}{5 - 2}$

$\Rightarrow \frac{PA}{AQ} = \frac{2}{3} \Rightarrow PA : AQ = 2 : 3$

Now by section formula x and y becomes as shown below

Since, P(6, -6) and Q(-4, -1)

$$\therefore x = \frac{mx_2 + nx_1}{m+n} = \frac{2(-4)+3\times 6}{2+3}$$

$$= \frac{-8+18}{5} = \frac{10}{5} = 2$$

$$y = \frac{my_2 + ny_1}{m+n} = \frac{2\times(-1)+3(-6)}{2+3}$$

$$= \frac{-2-18}{5} = \frac{-20}{5} = -4$$

Therefore Coordinates of A are (2, -4). As A lies on line segment joining the points P and Q so it must satisfy equation of line segment.

Therefore Substituting the value of x and y i.e; value of A (2,-4) in 3x + k(y + 1) =0

$\Rightarrow 3 \times 2 + k(-4 + 1) = 0 \Rightarrow 6 - 3k = 0$

$$\Rightarrow 3k = 6 \Rightarrow k = \frac{6}{3} = 2$$

OR

In equilateral $\triangle ABC$, coordinates of points A and B are (2,0) and (5,0) respectively.we have to find the co-ordinates of the other two vertices.

Let co-ordinates of C be (x, y)

Since $AC^2 = BC^2$ (sides of equilateral triangle)

$(x - 2)^2 + (y - 0)^2 = (x - 5)^2 + (y - 0)^2$

or, $x^2 + 4 - 4x + y^2 = x^2 + 25 - 10x + y^2$

or, 6x = 21

$$x = \frac{7}{2}$$

And $(x - 2)^2 + (y - 0)^2 = 9$

$$\text{or, } \left(\frac{7}{2} - 2\right)^2 + y^2 = 9$$

$$\text{or, } \frac{9}{4} + y^2 = 9$$

$$\text{or, } y^2 = \frac{27}{4} = \frac{3\sqrt{3}}{2}$$

$$\text{Hence, } C = \left(\frac{7}{2}, \frac{3\sqrt{3}}{2}\right)$$

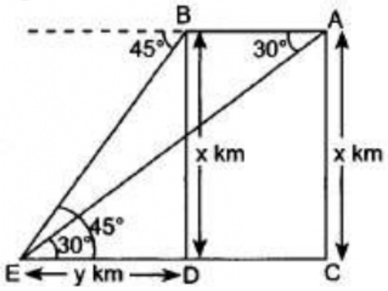
29. We have to find the greatest number that divides 445, 572 and 699 and leaves remainders of 4, 5 and 6 respectively. This means when the number divides 445, 572 and 699, it leaves remainders 4, 5 and 6. It means that

445 - 4 = 441,

572 - 5 = 567

and 699 - 6 = 693
are completely divisible by the required number.
For the highest number which divides the above numbers we need to calculate HCF of 441, 567 and 693 .
Therefore, the required number is the H.C.F. of 441, 567 and 693 Respectively.
First, consider 441 and 567.
By applying Euclid's division lemma, we get
$567 = 441 \times 1 + 126$
$441 = 126 \times 3 + 63$
$126 = 63 \times 2 + 0.$
Therefore, H.C.F. of 441 and 567 = 63
Now, consider 63 and 693
again we have to apply Euclid's division lemma, we get
$693 = 63 \times 11 + 0.$
Therefore, H.C.F. of 441, 567 and 693 is 63
Hence, the required number is 63. 63 is the highest number which divides 445,572 and 699 will leave 4,5 and 6 as remainder respectively.

30. Distance covered in 15 seconds $= AB$
Speed $= 1200\ km/hr.$



$$\therefore AB = 1200 \times \frac{15}{3600} = 5\text{km}$$

$AB = DC = 5km$
Let height = x km
In rt. $\triangle$BDE,

$$\frac{BD}{ED} = \tan 45^\circ \Rightarrow \frac{x}{y} = 1 \Rightarrow x = y$$

In rt. $\triangle$ACE,

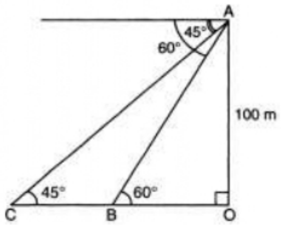
$$\frac{AC}{EC} = \tan 30^\circ \Rightarrow \frac{x}{y+5} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \frac{x}{x+5} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \sqrt{3}x = x + 5 \Rightarrow (\sqrt{3} - 1)x = 5$$

$$\therefore \quad x = \frac{5}{\sqrt{3}-1} = \frac{5(\sqrt{3}+1)}{2} = 6.83\text{km}$$

OR



In the given figure,

$\angle ACO = \angle CAX = 45°$

and $\angle ABO = \angle XAB = 60°$

Let A be a point and B, C be two objects.

In △ AOC, $\frac{AO}{CO} = \tan 45°$

$\Rightarrow \quad \frac{100}{\text{CO}} = 1$

$\Rightarrow \; CO = 100m$

Also in △ ABO, $\frac{AO}{OB} = \tan 60°$

$\Rightarrow \quad \frac{100}{OB} = \sqrt{3}$

$\Rightarrow \quad OB = \frac{100}{\sqrt{3}}$

$\therefore \quad BC = CO - OB = 100 - \frac{100}{\sqrt{3}}$

$= 100\left(1 - \frac{1}{\sqrt{3}}\right)\text{m}$

$100\frac{(\sqrt{3}-1)}{\sqrt{3}} = 100\frac{(\sqrt{3}-1)}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{100(3-\sqrt{3})}{3}\text{m}$

31.

Class interval	Mid value (x)	Frequency (f)	fx	Cumulative frequency
0 - 20	10	6	60	6
20 - 40	30	8	240	17
40 - 60	50	10	500	24
60 - 80	70	12	840	36
80 - 100	90	6	540	42
100 - 120	110	5	550	47

120 - 140	130	3	390	50
		N = 50	$\Sigma fx = 3120$	

Mean $= \frac{\Sigma fx}{N} = \frac{3120}{50} = 62.4$

We have,

N = 50

Then, $\frac{N}{2} = \frac{50}{2} = 25$

The cumulative frequency just greater than $\frac{N}{2}$ is 36, then the median class is 60 - 80

such that

l = 60, h = 80 - 60 = 20, f = 12, F = 24

$$\text{Median} = l + \frac{\frac{N}{2} - F}{f} \times h$$

$$= 60 + \frac{25 - 24}{12} \times 20$$

$$= 60 + \frac{20}{12}$$

= 60 + 1.67

= 61.67

Here the maximum frequency is 12, then the corresponding class 60 - 80 is the modal class

l = 60, h = 80 - 60 = 20, f = 12, $f_1 = 10$, $f_2 = 6$

$$\text{Mode} = l + \frac{f - f_1}{2f - f_1 - f_2} \times h$$

$$= 60 + \frac{12 - 10}{2 \times 12 - 10 - 6} \times 20$$

$$= 60 + \frac{40}{8}$$

= 65

Section D

32. i. By Elimination method

The given system of equation is

$\frac{x}{2} + \frac{2y}{3} = -1$...............(1)

$x - \frac{y}{3} = 3$....................(2)

Multiplying equation (2) by 2, we get

$$2x - \frac{2y}{3} = 6 \text{......................(3)}$$

Adding equation(1) and equation (2), we get

$$\frac{5}{2}x = 5 \Rightarrow x = \frac{5 \times 2}{5} \Rightarrow \quad x = 2$$

Substituting this value of x in equation(2), we get

$$2 - \frac{y}{3} = 3 \Rightarrow \frac{y}{3} = 2 - 3 = -1 \Rightarrow \quad y = -3$$

So, the solution of the given system of equation is

x = 2, y = -3

ii. By substitution method

The given system of equation is

$$\frac{x}{2} + \frac{2y}{3} = -1 \text{...............(1)}$$

$$x - \frac{y}{3} = 3 \text{....................(2)}$$

From equation (2),

$$x = \frac{y}{3} + 3 \text{....................(3)}$$

Substituting this value of x in (1),

$$\frac{1}{2}\left(\frac{y}{3} + 3\right) + \frac{2y}{3} = -1$$

$$\Rightarrow \quad \frac{y}{6} + \frac{3}{2} + \frac{2y}{3} = -1 \Rightarrow \quad \frac{5y}{6} = -1 - \frac{3}{2}$$

$$\Rightarrow \quad \frac{5y}{6} - -\frac{5}{2} \Rightarrow y = -3$$

Substituting this value of y in equation (3), we get

$$x = -\frac{3}{3} + 3 = -1 + 3 - 2$$

So, the solution of the given system of equations is x = 2, y = -3

Verification: Substituting x = 2, y = -3, we find that both the equation (1) and (2) are satisfied as shown below:

$$\frac{x}{2} + \frac{2y}{3} = \frac{2}{2} + \frac{2(-3)}{3} = 1 - 2 = -1$$

OR

Let the incomes of two persons be 11x and 7x.

And the expenditures of two persons be 9y and 5y

$\therefore$ $11x - 9y = 400$....(i)

$7x - 5y = 400$ ....(ii)

Multiplying (i) by 5 and (ii) by 9 and subtracting,

$$\begin{aligned} 55x - 45y &= 2{,}000 \\ 63x - 45y &= 3{,}600 \\ - \quad + \quad & \quad - \end{aligned}$$

$-8x = -1600$

$\therefore \; -8x = -1600$

$$x = \frac{-1{,}600}{-8} = 200$$

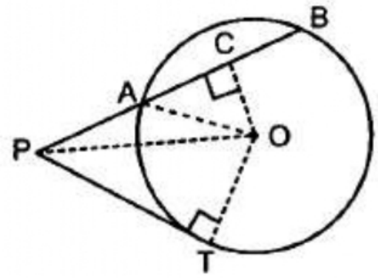
Therefore, Their monthly incomes are

$11 \times 200 = \text{Rs } 2200$

$7 \times 200 = \text{Rs}1400$

33. Given, PT is tangent to the circle at T.

PA = 4 cm and AB = 5 cm.



Construction: Draw OC $\perp$ AB and join OP, OT and OA.

Proof: In right $\triangle$OCP

$OP^2 = PC^2 + OC^2$

$OP^2 = [AP + AC]^2 + OC^2$

$$\mathbf{OP}^2 = \left[4 + \frac{1}{2}\mathbf{AB}\right]^2 + \mathbf{OC}^2 \; [OC \perp AB, AC = BC]$$

$$\Rightarrow OP^2 = \left(4 + \frac{5}{2}\right)^2 + OC^2$$

$$\Rightarrow OP^2 = \left(\frac{13}{2}\right)^2 + OC^2 \quad ..(i)$$

In right $\triangle$OCA,

$OA^2 = OC^2 + AC^2$

$OA^2 - AC^2 = OC^2$

$$OA^2 - \left(\frac{5}{2}\right)^2 = OC^2 \quad ..,(ii)$$

$\therefore$ eq (i) becomes.

$$OP^2 = \left(\frac{13}{2}\right)^2 + OA^2 - \left(\frac{5}{2}\right)^2$$

$$OP^2 = \frac{169}{4} - \frac{25}{4} + OA^2$$

$$OP^2 = \frac{144}{4} + OA^2$$

$\Rightarrow OP^2 = 36 + OA^2$ ...(iii)

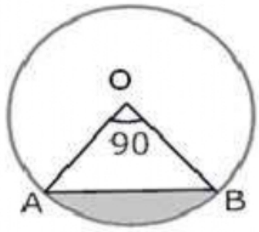
Also, $OP^2 = OT^2 + PT^2$ ...(iv)

from (iv) and (iii),

$PT^2 + OT^2 = 36 + OA^2$

$\Rightarrow PT^2 = 36$ [$\therefore$ OT = OA = radii]

$PT = 6cm.$

34. 

i. Area of minor sector $= \frac{\theta}{360}\pi r^2$

$= \frac{90}{360}(3.14)(10)^2$

$= \frac{1}{4} \times 3.14 \times 100$

$= \frac{314}{4}$

$= 78.50 = 78.5\ cm^2$

ii. Area of major sector = Area of circle - Area of minor sector

$= \pi(10)^2 - \frac{90}{360}\pi(10)^2 = 3.14\ (100) - \frac{1}{4}(3.14)\ (100)$

$= 314 - 78.50 = 235.5\ cm^2$

iii. We know that area of minor segment

= Area of minor sector OAB - Area of ΔOAB

$\because$ area of $\triangle OAB = \frac{1}{2}(OA)(OB)\sin\angle AOB$

$= \frac{1}{2}(OA)(OB)\left(\because \angle AOB = 90^\circ\right)$

Area of sector $= \frac{\theta}{360}\pi r^2$

$= \frac{1}{4}(3.14)\ (100) - 50 = 25(3.14) - 50 = 78.50 - 50 = 28.5\ cm^2$

iv. Area of major segment = Area of the circle - Area of minor segment

$= \pi(10)^2 - 28.5$

$= 100(3.14) - 28.5$

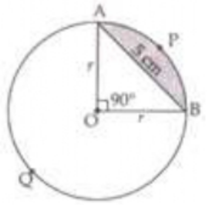
$= 314 - 28.5 = 285.5 \text{ cm}^2$

OR

Chord AB = 5 cm divides the circle into two segments minor segment APB and major segment AQB. We have to find out the difference in area of major and minor segment.

Here, we are given that $\theta = 90°$

Area of $\triangle OAB = \frac{1}{2}\text{Base} \times \text{Altitude} = \frac{1}{2}r \times r = \frac{1}{2}r^2$



Area of minor segment APB

$$= \frac{\pi r^2 \theta}{360°} - \text{Area of } \triangle AOB$$

$$= \frac{\pi r^2 90°}{360°} - \frac{1}{2}r^2$$

$$\Rightarrow \text{Area of minor segment} = \left(\frac{\pi r^2}{4} - \frac{r^2}{2}\right) \ldots(i)$$

Area of major segment AQB = Area of circle – Area of minor segment

$$= \pi r^2 - \left[\frac{\pi r^2}{4} - \frac{r^2}{2}\right]$$

$$\Rightarrow \text{Area of major segment AQB} = \left[\frac{3}{4}\pi r^2 + \frac{r^2}{2}\right] \ldots(ii)$$

Difference between areas of major and minor segment

$$= \left(\frac{3}{4}\pi r^2 + \frac{r^2}{2}\right) - \left(\frac{\pi r^2}{4} - \frac{r^2}{2}\right)$$

$$= \frac{3}{4}\pi r^2 + \frac{r^2}{2} - \frac{\pi r^2}{4} + \frac{r^2}{2}$$

$$\Rightarrow \text{Required area} = \frac{2}{4}\pi r^2 + r^2 = \frac{1}{2}\pi r^2 + r^2$$

In right $\triangle OAB$,

$r^2 + r^2 = AB^2$

$\Rightarrow 2r^2 = 5^2$

$\Rightarrow r^2 = \frac{25}{2}$

Therefore, required area $= \left[\frac{1}{2}\pi \times \frac{25}{2} + \frac{25}{2}\right] = \left[\frac{25}{4}\pi + \frac{25}{2}\right] cm^2$

35. The houses are numbered consecutively from 1 to 49.

1, 2, 3....................(x - 1), x, (x+1).........49

Sum of number of houses preceding x numbered house = Sum of number following x

Sum of number of houses preceding x numbered house=

$S_1 = \frac{x-1}{2} \times (1 + x - 1) = \frac{x(x-1)}{2}$ - - - - - - - - (1)

Sum of number following x=

$S_2 = (1 + 2 + 3 + ...49) - \frac{x}{2} \times (x + 1)$

$= \frac{49 \times 50}{2} - \frac{x^2+x}{2} = \frac{2450-x^2-x}{2}$ - - - - - - (2)

As $S_1 = S_2$

$\frac{2450-x^2-x}{2} = \frac{x(x-1)}{2}$

$2450 - x^2 - x = x^2 - x$

$2x^2 = 2450$

$x^2 = 1225$

$x = 35$

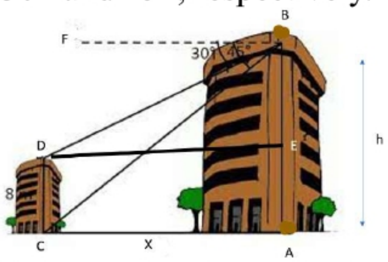
Hence sum of numbers of houses proceeding the house numbered 35 is equal to sum of the numbers of houses following 35

Section E

36. **Read the text carefully and answer the questions:**

Basant and Vinod lives in a housing society in Dwarka, New Delhi. There are two building in their housing society. The first building is 8 meter tall. One day, both of them were just trying to guess the height of the other multi-storeyed building. Vinod said that it might be a 45 degree angle from the bottom of our building to the top of multi-storeyed building so the height of the building and distance from our building to this multi-storeyed building will be same. Then, both of them decided to estimate it using some trigonometric tools. Let's assume that the first angles of depression of the top and bottom of an 8 m tall building from top of a multi-storeyed building are

30^{o} and 45^{o}, respectively.



(i) Let h is height of big building, here as per the diagram.

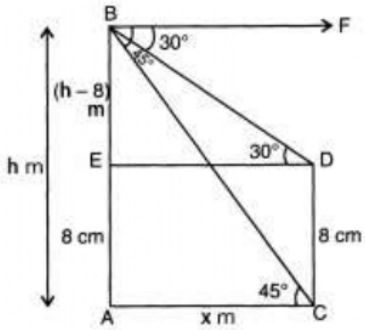
AE = CD = 8 m (Given)

BE = AB - AE = (h - 8) m

Let AC = DE = x

Also, $\angle FBD = \angle BDE = 30^{\circ}$

$\angle FBC = \angle BCA = 45^{\circ}$



In △ACB, $\angle A = 90^{\circ}$

$\tan 45^{\circ} = \frac{AB}{AC}$

$\Rightarrow$ x = h, ...(i)

In △ BDE, $\angle E = 90^{\circ}$

$\tan 30^{\circ} = \frac{BE}{ED}$

$\Rightarrow x = \sqrt{3}(h-8)$.(ii)

From (i) and (ii), we get

$h = \sqrt{3}h - 8\sqrt{3}$

$h(\sqrt{3} - 1) = 8\sqrt{3}$

$h = \frac{8\sqrt{3}}{\sqrt{3}-1} = \frac{8\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$= \frac{1}{2} \times (24 + 8\sqrt{3}) = \frac{1}{2} \times (24 + 13.84) = 18.92$ m

(ii) Let h is height of big building, here as per the diagram.

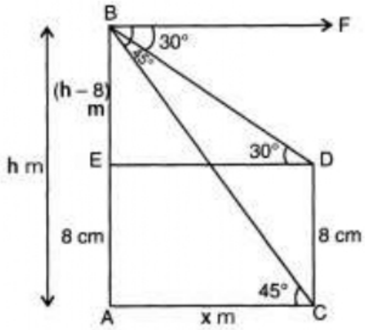
AE = CD = 8 m (Given)

BE = AB - AE = (h - 8) m

Let AC = DE = x

Also, $\angle FBD = \angle BDE = 30°$

$\angle FBC = \angle BCA = 45°$



In △ACB, $\angle A = 90°$

$$\tan 45° = \frac{AB}{AC}$$

$\Rightarrow$ x = h, ...(i)

In △ BDE, $\angle E = 90°$

$$\tan 30° = \frac{BE}{ED}$$

$\Rightarrow x = \sqrt{3}(h-8)$.(ii)

From (i) and (ii), we get

$h = \sqrt{3}h - 8\sqrt{3}$

$h(\sqrt{3} - 1) = 8\sqrt{3}$

$$h = \frac{8\sqrt{3}}{\sqrt{3}-1} = \frac{8\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{1}{2} \times (24 + 8\sqrt{3}) = \frac{1}{2} \times (24 + 13.84) = 18.92 \text{ m}$$

Hence height of the multistory building is 18.92 m and the distance between two buildings is 18.92 m.

(iii)In △ABC

$$\sin 45° = \frac{AB}{BC}$$

$$\Rightarrow BC = \frac{AB}{\sin 45^0}$$

$$\Rightarrow BC = \frac{18.92}{\frac{1}{\sqrt{2}}}$$

$\Rightarrow$ BC = 26.76 m

Hence the distance between top of multistoried building and bottom of first building is 26.76 m.

OR

In $\triangle BDE$

$$\cos 30^o = \frac{ED}{BD}$$

$$\Rightarrow BD = \frac{ED}{\cos 30^0}$$

$$\Rightarrow BD = \frac{\frac{8\sqrt{3}}{\sqrt{3}-1}}{\frac{\sqrt{3}}{2}} = \frac{16}{\sqrt{3}-1}$$

$$\Rightarrow BD = 8\left(\sqrt{3}+1\right) = 21.86 \text{ m}$$

Hence, the distance between top of multistoried building and top of first building is 21.86 m.

37. **Read the text carefully and answer the questions:**

Elpis Technology is a TV manufacturer company. It produces smart TV sets not only for the Indian market but also exports them to many foreign countries. Their TV sets have been in demand every time but due to the Covid-19 pandemic, they are not getting sufficient spare parts, especially chips to accelerate the production. They have to work in a limited capacity due to the lack of raw materials.

(i) Since the production increases uniformly by a fixed number every year. Therefore, the sequence formed by the production in different years is an A.P. Let a be the first term and d be the common difference of the A.P. formed i.e., 'a' denotes the production in the first year and d denotes the number of units by which the production increases every year.

We have, $a_3 = 600$ and

$a_3 = 600$

$\Rightarrow$ $600 = a + 2d$

$\Rightarrow$ $a = 600 - 2d$ ...(i)

$\Rightarrow$ $a_7 = 700$

$\Rightarrow$ $a_7 = 700$

$\Rightarrow$ $700 = a + 6d$

$\Rightarrow$ $a = 700 - 6d$ ...(ii)

From (i) and (ii)

$600 - 2d = 700 - 6d$

$\Rightarrow$ $4d = 100$

$\Rightarrow$ $d = 25$

(ii) Since the production increases uniformly by a fixed number every year. Therefore, the sequence formed by the production in different years is an A.P. Let a be the first term and d be the common difference of the A.P. formed i.e., 'a' denotes the production in the first year and d denotes the number of units by which the production increases every year.

We know that first term = a = 550 and common difference = d = 25

$a_n = 1000$

$\Rightarrow 1000 = a + (n - 1)d$

$\Rightarrow 1000 = 550 + 25n - 25$

$\Rightarrow 1000 - 550 + 25 = 25n$

$\Rightarrow 475 = 25n$

$\Rightarrow n = \frac{475}{25} = 19$

(iii) Since the production increases uniformly by a fixed number every year. Therefore, the sequence formed by the production in different years is an A.P. Let a be the first term and d be the common difference of the A.P. formed i.e., 'a' denotes the production in the first year and d denotes the number of units by which the production increases every year.

The production in the 10th term is given by a_{10}. Therefore, production in the 10th year = $a_{10} = a + 9d = 550 + 9 \times 25 = 775$. So, production in 10th year is of 775 TV sets.

OR

Since the production increases uniformly by a fixed number every year. Therefore, the sequence formed by the production in different years is an A.P. Let a be the first term and d be the common difference of the A.P. formed i.e., 'a' denotes the production in the first year and d denotes the number of units by which the production increases every year.

Total production in 7 years = Sum of 7 terms of the A.P. with first term a (= 550) and d (= 25).

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow S_7 = \frac{7}{2}[2 \times 550 + (7-1)25]$$

$$\Rightarrow S_7 = \frac{7}{2}[2 \times 550 + (6) \times 25]$$

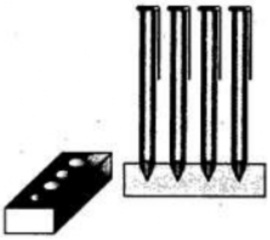
$$\Rightarrow S_7 = \frac{7}{2}[1100 + 150]$$

$$\Rightarrow S_7 = 4375$$

38. **Read the text carefully and answer the questions:**

A carpenter in the small town of Bareilly used to make and sell different kinds of wood items like a rectangular box, cylindrical pen stand, and cuboidal pen stand. One day a student came to his shop and asked him to make a pen stand with the dimensions as follows:

A pen stand should be in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid should be 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm.



(i) Volume of the cuboid

$= 15 \times 10 \times 3.5 = 525 cm^3$

(ii) Volume of a conical depression

$= \frac{1}{3}\pi(0.5)^2(1.4)$

$= \frac{1}{3} \times \frac{22}{7} \times 0.25 \times \frac{14}{10} = \frac{11}{30} cm^3$

$\therefore$ Volume of four conical depressions

$= 4 \times \frac{11}{30} cm^3 = \frac{22}{15} cm^3 = 1.47 cm^3$

(iii) $\therefore$ Volume of the wood in the entire stand = volume of cuboid - volume of 4 conical depressions

$= 525 - 1.47 = 523.53 cm^3$

OR

Cost of wood per cm^3 = ₹10

Total cost of making pen stand = 10 × 523.53 = ₹5235.3